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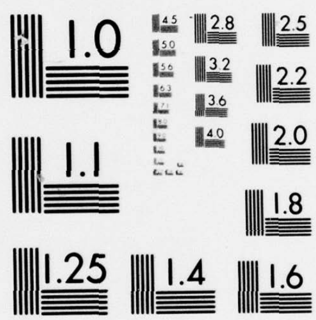
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COMPARISON OF THE EFFECT OF STRUCTURAL COUPLING PARAMETERS ON FLAP-LAG FORCED RESPONSE AND STABILITY OF A HELICOPTER ROTOR BLADE IN FORWARD FLIGHT (U)

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INTRODUCTION

Compared to the conventional airplane, the present day helicopter is a modern invention. Although Leonardo da Vinci sketched a man-carrying vehicle with rotating wings around the year 1500, only after World War II has this versatile aircraft come into general use throughout the world. During the last decade the helicopter has become one of man's most useful tools in the air, demonstrating its versatility in both civil and military roles. The reasons for the long delay in realizing Leonardo's dream have not been for want of trying.

If we consider the first flight of de la Cierva's autogyros in 1925-26 as the birth of rotary-wing aviation, then the "childhood diseases" of the helicopter were mainly solved in the first decade (1926-1936). These initial problems were aerodynamic in nature and consisted of balancing, controllability, and stability. More serious deficiencies became apparent, however, when the first series-produced machines appeared and were placed in service. These problems were dynamic in nature such as fatigue due to insufficient dynamic strength of certain structural members and degradation of crew and passenger efficiency due to unacceptable vibration levels (Reference 1). While sufficient control of these deficiencies has been achieved to allow the helicopter to progress to its present state, they remain as a major obstacle in complete maturity phase development of the helicopter.

The loads on the helicopter are quite different from a conventional propeller driven aircraft. The propeller operates practically in an axial flow and, like an engine, sets up no noticeable variable stresses in the structural members. Therefore, it is only during take-off, landing, and maneuvers that appreciable dynamic loads are induced on the aircraft structure. Since these loads usually represent relatively few load cycles during the lifetime of the aircraft, one can generally speak of repeated static loads. The helicopter's main structural members are loaded dynamically, the number of loadings often exceeding tens of millions of cycles during its lifetime. The cause of this dynamic loading is due primarily to the asymmetric airflow past the main rotor as it rotates and simultaneously advances (Reference 1).

In forward flight, when the main rotor provides both lift and propulsive force for the helicopter, the flow over the blades is asymmetric due to the velocity differential over the advancing and retreating blades (Figure 1). Rotor control is obtained by "cyclic pitch" change, which is the name given to the first harmonic variation applied to the blade pitch angle as it rotates. Since the relative air velocity over the blade also has a first harmonic variation and since aerodynamic forces are proportional to the square of the relative velocity, we may expect to find at least three harmonics in the force fluctuations acting on the blades. This would be true if the airflow through the rotor were uniform, however, due to the proximity of the rotor to its own vortex wake, which is swept backwards under the rotor disc, the flow is far from uniform, and velocity fluctuations are induced which give rise to very many harmonics of blade loading. The aerodynamic characteristics of a rotor in forward flight give rise to shear forces and moments at the blade root which are then transmitted to the rotor hub where they are combined and sent through the rotor shaft into the airframe. The dynamics complexity of the helicopter can be ascertained from the cartoon in Figure 2.

During the design of a helicopter, accurate prediction of main rotor blade and hub oscillatory loading is important for fatigue, vibration, and forward flight performance. Critical dynamic components are designed for fatigue based on these predicted loads. Vibration characteristics of the airframe and the need for vibration reduction devices are determined from these loads. Since vibratory response and rotor loads usually determine limiting speed and load factor operational envelopes, the predicted loads greatly influence forward flight performance. Recent Army helicopter development programs, Utility Tactical Transport Aircraft System (UTTAS) and Advanced Attack Helicopter (AAH), have revealed that these loads cannot be predicted accurately (Reference 2). A similar conclusion was

drawn from a comparison study (Reference 3) where the helicopter industry's major state-of-the-art rotor loads analyses were independently exercised on an identical hypothetical helicopter problem. The comparison of results illustrated significant differences, particularly in structural dynamic modeling, between the various analyses. A strong recommendation as a result of this study was to conduct computer experiments to study specific isolated aspects of the solution methods and structural dynamics (Reference 3). It was the results of this study and the UTTAS/AAH Programs that provided the impetus for this research.

ROTOR BLADE AZIMUTHS AND VELOCITIES

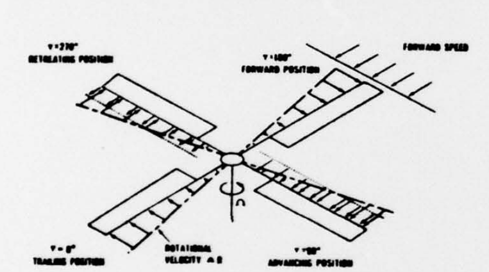

FIGURE 1

DYNAMICS COMPLEXITY

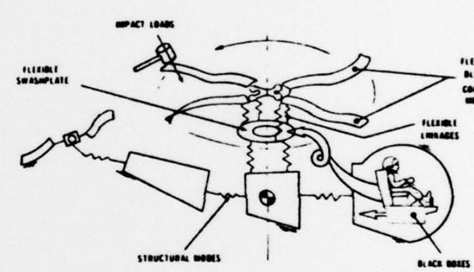

FIGURE 2

THE MATHEMATICAL MODEL

Description

The model is illustrated in Figure 3. It consists of a slender rigid blade, hinged at the center of rotation, with spring restraint at the hinge. The orientation of the flap (out-of-plane) and lag (in plane) restraint springs which are parallel and perpendicular to the blade chord line, respectively, simulate the elastic coupling characteristics of the actual elastic blade. The spring stiffnesses are chosen so that the uncoupled rotating flap and lag natural frequencies coincide with the corresponding first mode rotating natural frequencies of the elastic blade. This allows the model to represent a hingeless rotor treated by virtual hinges or a fully hinged rotor (Reference 4).

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The model chosen has been used to evaluate rotor aeroelastic stability and can include the incorporation of flap-lag elastic coupling, pitch-flap and pitch-lag coupling, and a blade lag damper. These were some of the fundamental dynamic mechanisms found important in the stability studies of References 4 and 5. Their importance with regards to forced response is a major objective sought in this research.

CENTRALLY HINGED, SPRING RESTRAINED, RIGID BLADE REPRESENTATION

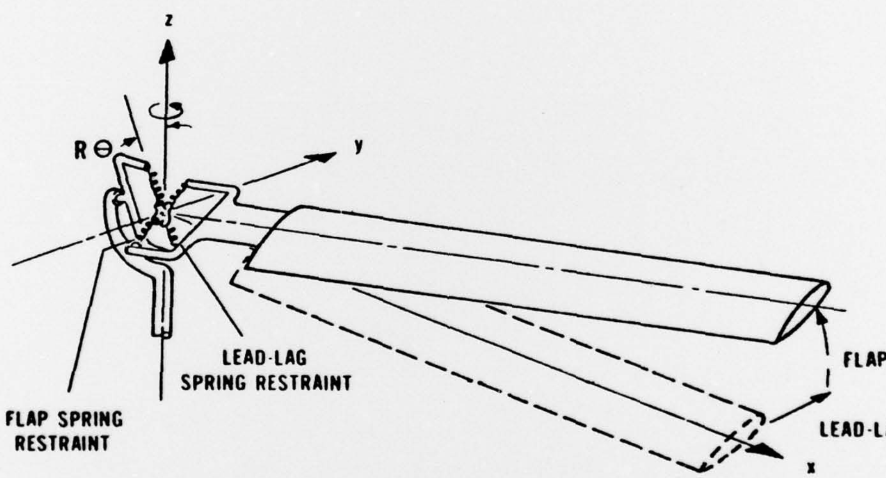

FIGURE 3

To include flap-lag structural coupling and be able to vary it to determine its effect on blade vibrations, the parameter R is defined. Geometrically, it is defined as the ratio of the blade pitch angle inboard and outboard of the pitch bearing. Physically, it represents how the flap-lag structural coupling is dependent on the relative stiffness of the blade segments inboard and outboard of the pitch bearing. This is because the principal elastic axes of the outboard segment rotate through an angle as the blade pitch varies while the inboard segment principal axes do not. The method of implementation is accomplished by replacing θ by $R\theta$ in the structural terms in the flap-lag equations while the mass and inertial terms

are unchanged. Thus, when R=1, the original equations are retained, but as R is reduced to zero, the flap-lag structural coupling terms diminish and eventually vanish (Reference 4).

Torsion is included in a quasi-steady manner as pitch-flap and pitch-lag coupling. This is accomplished by defining the pitch angle, θ, as:

$$\theta = \theta_o + \theta_s \sin\psi + \theta_c \cos\psi + \theta_\beta\, \delta\beta + \theta_\zeta \delta\zeta$$

Pitch-flap coupling, θ_β, is intentionally built into some rotors to reduce the flapping motion. If negative θ_β is used, then as the blade flaps upwardly the pitch is decreased, which reduces the aerodynamic forces on the blade and the flapping motion is somewhat suppressed. Pitch-lag coupling, θ_ζ, is sometimes used in a similar way to reduce the lagging motion.

As stated previously, the model has been used to evaluate aeroelastic stability. These studies, (References 3 and 5), have shown the model to give quite reasonable accuracy at low advance ratios when compared with more complicated models. Since all pure helicopters currently developed operate at a $\mu < 0.5$ and the aeroelastic stability of a rotor is considered more sensitive to system parameters than forced response the model is believed to be an adequate representation for parametric studies.

<u>Equations of Motion</u>

The general procedure followed was to derive the nonlinear flap-lag equations of a helicopter rotor in forward flight. The left hand side of the equations was made linear by considering small perturbation motions about a periodic equilibrium position (trimmed condition) of the nonlinear system. The nonlinear terms were consolidated on the right hand side to form the following flap-lag perturbation equations of motion:

$$\begin{Bmatrix} \delta\ddot{\beta} \\ \delta\ddot{\zeta} \end{Bmatrix} + \left[C(\psi)\right] \begin{Bmatrix} \delta\dot{\beta} \\ \delta\dot{\zeta} \end{Bmatrix} + \left[K(\psi)\right] \begin{Bmatrix} \delta\beta \\ \delta\zeta \end{Bmatrix} = \left[L(\psi)\right] \begin{Bmatrix} \hat{\bar{\beta}} \\ \bar{\beta} \end{Bmatrix} + \left[M(\psi)\right] \begin{Bmatrix} \bar{\theta} \\ \bar{\Phi} \end{Bmatrix} + \left[N(\psi)\right] \begin{Bmatrix} \ddot{\bar{\beta}} \\ \frac{Cdo}{a} \end{Bmatrix}$$

There are two forms of periodic coefficients in the equations. The explicit periodic coefficients are of the form $\mu \sin \psi$ or $\mu \cos \psi$. These types are only associated with aerodynamic terms. The second type are implicit periodic coefficients that result from the fact that $\bar{\theta}$ and $\bar{\beta}$ may have cyclic components. These types are

found in both structural and aerodynamic terms. A damping term is included in the $C(\psi)$ matrix to account for the lead-lag damper included on many rotors, particularly hinged rotors. These equations are put into state variable format to reduce them to first order so that an efficient computer library differential equation solver can be utilized.

METHOD OF SOLUTION

Eigenvalue and Modal Decoupling Analysis

The method used to solve the linearized flap-lag equations with periodic coefficients is called an eigenvalue and modal decoupling analysis. It takes advantage of Floquet Theory with its application to linear systems with time varying coefficients. It is unique in that it extends the Floquet Transition Matrix (FTM) Method (Reference 6) to include the calculation of forced response. The eigenvalue and modal decoupling method is summarized in Figure 4. Once the flap-lag equations have been put into state variable format, the state transition matrix and FTM are obtained by step-wise integration over one period. This period consists of one rotor revolution from time zero to 2π. Once the FTM, $\Phi(T)$, is obtained it can be shown by application of the Floquet-Liapunov Theorem (Reference 6) that the problem of stability reduces to solution of an eigenvalue problem. Stability is determined from the real part of the complex eigenvalue, $\Lambda = \eta + i\omega$. Therefore, when η is negative the system is stable, with $\eta = 0$ representing the stability limit or boundary.

The extension of the FTM Method to determine forced response requires considerable manipulation although the mathematics are relatively simple. The eigenvectors of the FTM and the state transition matrix at each integration step increment must be obtained and saved. This is necessary in order that the characteristic functions, $A(t)$, can be calculated and used as a variable substitution to decouple the system of equations. This procedure is completely analogous to the modal analysis method (Reference 7) used to uncouple a system of equations with constant coefficients. Hence, the name eigenvalue and modal decoupling analysis is given to the method of solution.

Hub Shears and Moments

The information obtained from the analysis are the flap and lag displacements and velocities. These quantities are combined in the time domain to obtain flap and lag shears and moments in both the rotating and non-rotating systems. Once calculated the shears and

moments are harmonically analyzed through Fourier Series expansion to determine the relative strengths of the rotor harmonics. In the rotating system the primary concern is fatigue of root end and hub components so that the first harmonic (1/REV) is the major oscillatory source. In the fixed system, the rotor acts as a filter and only allows rotor harmonics that are integral multiples of the number of blades to be transmitted. Therefore, for a two bladed helicopter only the even harmonics, 2/REV, 4/REV, etc., are of major concern in the fixed system. The root shears and moments for each blade in the rotating coordinate system are expressed as the integrals of the blade aerodynamic and inertial loading. The single bladed results in the fixed system are valid for rotors with any number of blades so long as the appropriate solution harmonics are set to zero. Therefore, if a three bladed rotor is being analyzed only the 3/REV loads are of interest in the fixed system.

EIGENVALUE AND MODAL DECOUPLING ANALYSIS

Step	Description
1	Put Flap-Lag Equations into State Variable Format: $\dot{X}(t) + D(t)X(t) = f(t)$
2	Determine the State Transition Matrix, $\phi(t)$, and Floquet Transition Matrix, $Q = \phi(T)$, By Numerical Integration Over One Period. The Solution Can Be Written as: $X(t) = \phi(t)\, X(0)$
3	Find Eigenvalues and Eigenvectors of FTM, Q. Floquet's Theorem States that a System of Equations Having Periodic Coefficients Has Transient Solutions of the Form: $X(t) = A(t) \left[\diagdown e^{\Lambda t}\diagdown\right] \{\alpha\}$. At t=0: $\{\alpha\} = A(0)^{-1} X(0)$
4	Using 2 and 3 an Eigenvalue Problem is Developed: $A(0)^{-1} Q\, A(0) = \left[\diagdown e^{\Lambda t}\diagdown\right]$. The Characteristic Functions, A(t), Can Be Obtained From: $A(t) = \phi(t)A(0) \left[\diagdown e^{-\Lambda t}\diagdown\right]$
5	Through Variable Substitution, $X(t) = A(t)Y(t)$, the Equations in 1 Can Be Decoupled to Obtain: $\dot{Y}(t) - \left[\diagdown \Lambda \diagdown\right] Y(t) = g(t)$
6	The Uncoupled Response, Y(t), Can Be Obtained From Complex Fourier Series Expansion. The Coupled Response X(t), Can Be Obtained From Matrix Multiplication.

7 Convert X(t) Information in Time Domain to Vibratory Shears and Moments. Fourier Analyze Shears and Moments to Obtain Various Harmonic Components.

FIGURE 4

RESULTS AND DISCUSSION

As with any new analytical method correlation with other proven analyses or measured data must be obtained for validation. The hypothetical rotor of Reference 3 was used for this purpose. The input parameters for this articulated (hinged) rotor and for a hingeless rotor are presented in Figure 5.

ROTOR BLADE INPUT PARAMETERS

Type	p	ω_ζ	γ	σ	C_T	C_{do}	R	b	c	β_{pc}	θ_β	θ_ζ	Lag Damper
Hinged	1.031	0.25	7.5	.07	.0063	.01	25'	3	1.83	0	0	0	.1536
Hingeless	1.136	0.75	9.3	.1	.0062	.01	24.5'	4	1.92	.06	.15	0	0

FIGURE 5

The first rotor is typical of hinged rotors on current operational helicopters. It includes a lag damper, as do most hinged rotors, to insure freedom from mechanical instability, usually called ground resonance. Since the rotor has three blades the third harmonic (3/REV) is the largest source of vibration in the fixed system. However, this 3/REV vibration is caused by 2/REV, 3/REV, and 4/REV shears and moments in the rotating system. The 1/REV forces in the rotating system are most important when considering blade and hub component fatigue lives. The second rotor is called hingeless because it does not have a mechanical hinge to allow flapping and lagging motion as does the hinged rotor. The term "soft inplane" hingeless rotor is often applied because the inplane (lag) frequency is less than the rotor speed (1/REV). This rotor includes β_{pc} and θ_β in its baseline configuration. Since the rotor has four blades the first five rotor harmonics can be considered critical in the rotating system. This rotor is very similar to one of the UTTAS candidate helicopters.

Hinged Rotor Results

Calculated hub vertical and horizontal shears are compared in Figures 6 and 7 with the results of Reference 3. While significant differences exist, the eigenvalue and modal decoupling analysis

results, labeled FL for flap-lag, compares favorably with the much more sophisticated methods.

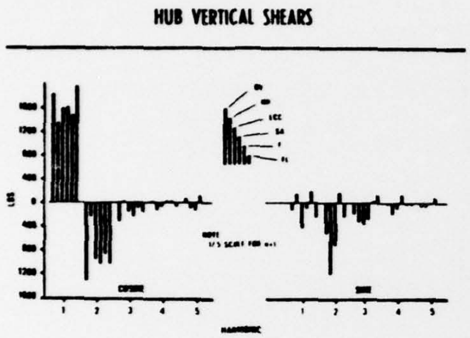

FIGURE 6

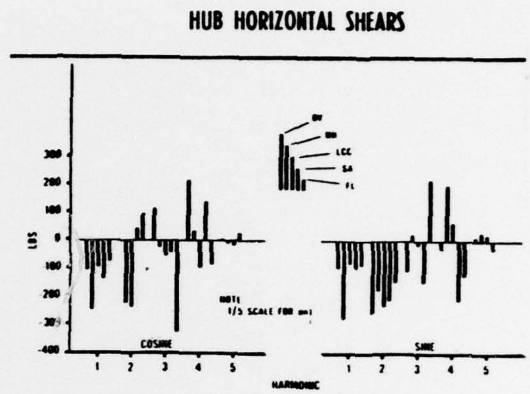

FIGURE 7

Blade lag dampers are considered essential for hinged rotors to insure freedom from mechanical instability. Coalescense between the blade lag motion and the helicopter rolling or pitching motion is the source of this instability. Since the lag motion is lightly damped a mechanical damper is added. Figures 8 and 9 illustrate the effect a mechanical damper setting has on lag damping and lag moments. It is not unusual for an operating hinged helicopter to have a 25% critical damping setting. As can be seen from Figure 9 this damper setting can greatly increase the lag moments.

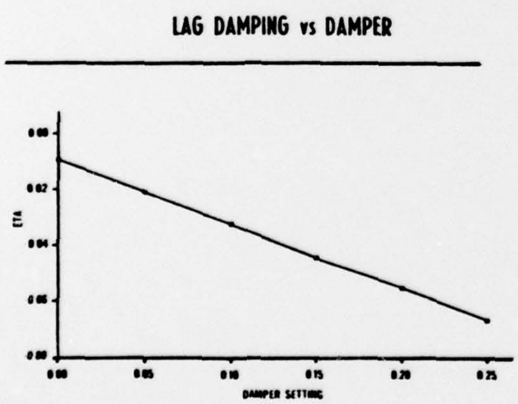

FIGURE 8

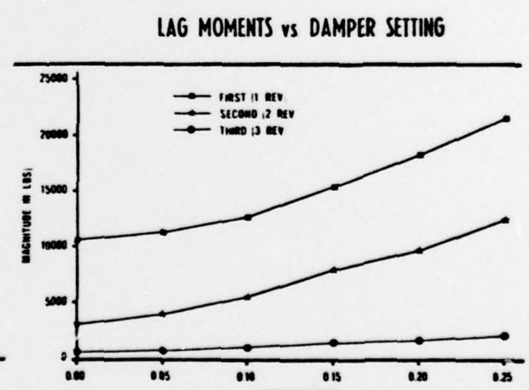

FIGURE 9

9

On hinged rotors pitch-lag and pitch-flap coupling are due to mechanical linkages which will cause the blade to pitch whenever there is a change in flap or lag angle. The variation of flap and lag damping with pitch-flap coupling is presented in Figure 10. The effect of pitch-flap coupling on lag shears is illustrated in Figure 11. It appears that negative pitch-flap coupling can be used effectively to reduce blade loading and airframe vibration (Figure 11) without jeopardizing stability (Figure 10). The contrary is true for positive pitch-flap coupling which results in a rapid decrease in flap damping above 0.4.

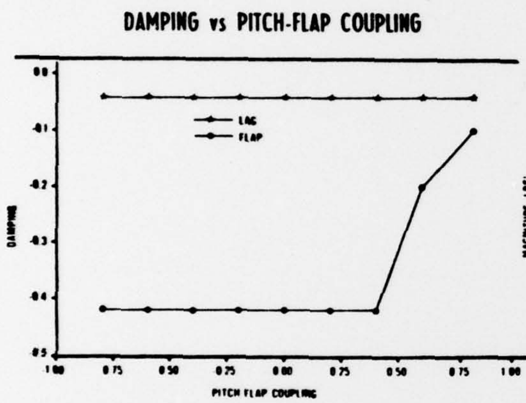

FIGURE 10

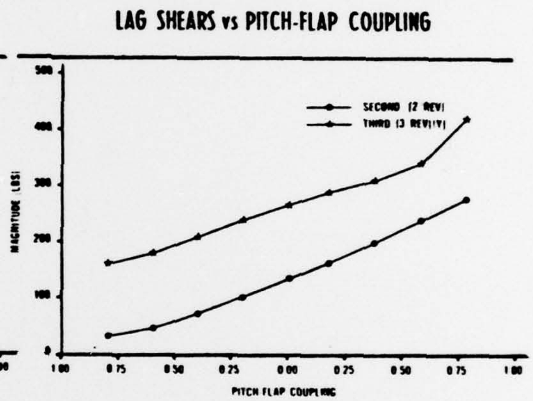

FIGURE 11

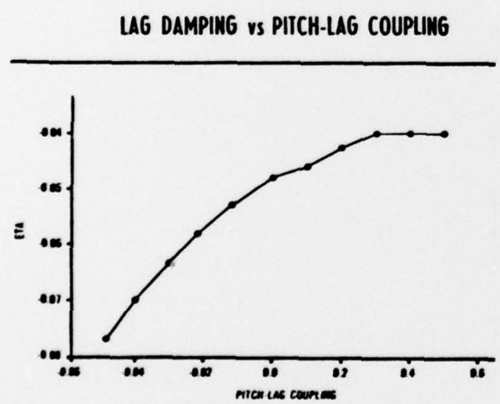

FIGURE 12

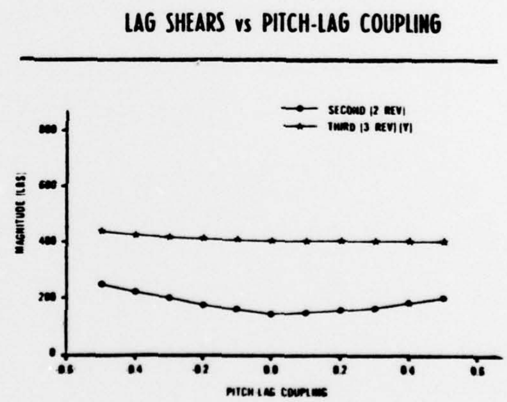

FIGURE 13

Pitch-lag coupling effects are illustrated in Figures 12 and 13. It appears that little benefit is gained with respect to lag

10

shears. Lag damping varies greatly with pitch-lag coupling. The flattening out of the damping curve in . gure 14 for positive pitch-lag coupling is due to the 15% critical damping setting for the baseline configuration (Figure 5).

Hingeless Rotor Results

Hingeless rotors are attractive because they increase the helicopter's control power, simplify the rotor mechanically, and reduce somewhat the hubweight and aerodynamic drag. With their hub cantilever boundary conditions they do complicate structural dynamic coupling and transmit large vibratory rotor loads into the airframe. Flap-lag elastic coupling is an important consideration on hingeless rotors and an area that is not well understood. Figures 14 and 15 illustrate the effect of flap-lag elastic coupling, R, on lag damping and lag shears, respectively. It can be seen in Figure 14 that small values of R greatly reduce lag damping but that the trend is reversed for larger values of R. With respect to lag shears; the 1/REV shears are greatly increased for small values of R and are relatively constant for larger values.

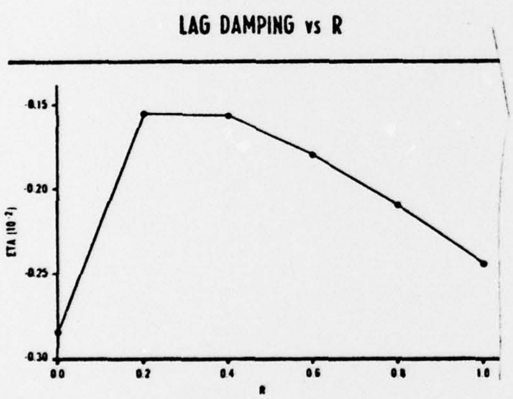

FIGURE 14

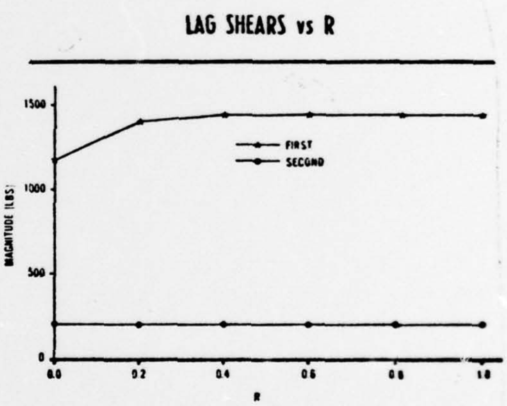

FIGURE 15

The variations with respect to pitch-flap and pitch-lag coupling are similar to the trends found for the hinged rotor. Figure 16 illustrates the reduction in flap damping with positive pitch-flap coupling. For a positive value of 1 the flap damping is reduced to almost zero, the stability boundary. For positive pitch-lag coupling, Figure 17, the lag damping becomes positive, indicating an instability. If no lag damper had been installed on the hinged rotor a similar trend would have been expected.

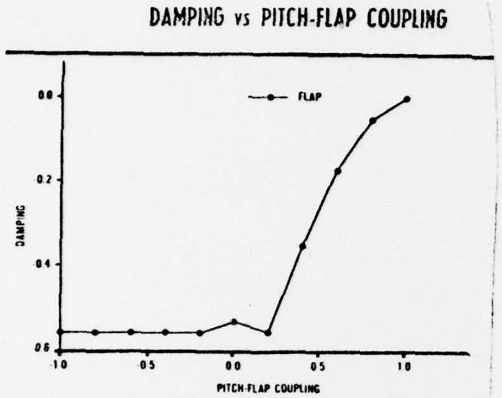

FIGURE 16

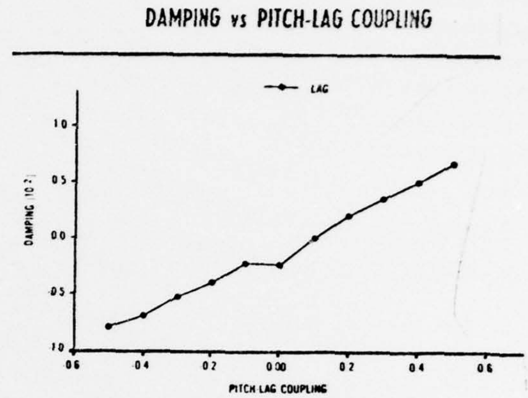

FIGURE 17

CONCLUSIONS AND RECOMMENDATIONS

An eigenvalue and modal decoupling method to predict helicopter rotor stability and forced response has been successfully developed. The advantages of this method are:

1. Stability and forced response are obtained from the same method; thus allowing direct comparison of parametric effects;

2. Only one rotor revolution of numerical integration for an initial condition of unity imposed on each degree of freedom is necessary to define the Floquet Transition Matrix, therefore, the ambiguous interpretation of time history response data over many rotor revolutions and the separation of transient and forced response is no longer necessary; and

3. Additional degrees of freedom such as hub and airframe motions, inflow feedback, etc. in both the fixed and rotating systems can be included since Floquet theory is applicable to equations with periodic coefficients.

The method has been applied to a hinged and hingeless rotor to compare the effect of structural coupling parameters on stability and forced response in forward flight. Significant conclusions drawn from this application are:

1. Pitch-flap coupling has a powerful influence on stability and forced response. Negative coupling reduces lag shears considerably

without jeopardizing stability, while positive coupling produces an opposite effect. This trend was similar for both rotors.

2. Pitch-lag coupling has little influence on forced response but significantly alters lag damping. The use of negative coupling resulted in increased lag damping for both rotors. Positive coupling reduced lag damping for the hinged rotor with lag damper and produced an instability on the hingeless rotor.

3. Variation in lag damper setting on the hinged rotor resulted in significant increases in lag moments and, naturally, increased lag damping.

4. The elastic coupling parameter, R, altered greatly the lag damping on the hingeless rotor. For low values of R the damping decreased drastically but increased again for larger R values. Low values of R also significantly increased the first harmonic lag moments.

The current research has been directed toward developing an alternative solution method to the blade stability and response problem. For this reason a relatively simple mathematical model was used to study the effects of structural coupling parameters. Further research should include:

1. More case studies with the current model to gain a better understanding of the structural coupling parameters in forward flight. Other rotor configurations, such as "stiff inplane" hingeless, should be included.

2. The method should be applied to an elastic blade model to gain a better understanding of higher harmonic interaction.

3. Correlation with wind tunnel and flight measurements should be made to further validate the model and method.

NOTATIONS

a	=	slope of lift curve
b	=	number of blades
c	=	blade chord, ft.
c_{do}	=	blade profile drag
$C(\psi)$	=	damping matrix
C_T	=	thrust coefficient
$K(\psi)$	=	stiffness matrix
p	=	nondim. rot. flapping frequency at $\theta=0$, $p=(1+\omega_\beta^2)^{\frac{1}{2}}$
r	=	blade radial coordinate, ft.
$\bar{r}$	=	nondim. coordinate r/R
R	=	blade radius, ft., or elastic coupling parameter
t	=	time, sec
T	=	rotor thrust
V_i	=	uniform induced velocity in negative Z direction, fps
V_x, V_z	=	components of helicopter speed in negative X and Z directions, respectively, fps
X, Y, Z, x, y, z	=	aircraft coordinates and rotating blade coordinates respectively
$\bar{X}$, $\bar{Y}$	=	Forward and Lateral Shears respectively
$L(\psi)$	=	forcing coefficient matrix
$M(\psi)$	=	forcing coefficient matrix
$N(\psi)$	=	forcing coefficient matrix

β	=	flap angle, pos up, rad
$\bar{\beta}$	=	equilibrium flap angle, $\beta_o + \beta_s S\psi + \beta_c C\psi$, rad
β_{pc}	=	precone angle, rad
α	=	Locke number, $\rho a c R^4/I$
$\delta\beta$, $\delta\zeta$	=	perturbation flap and lag angles, rad
ζ	=	lead-lag angle, pos. fwd, rad
$\bar{\zeta}$	=	equilibrium lead-lag angle, rad
η	=	neg real portion of lead-lag eigenvalue
θ	=	pitch angle $\theta + \theta_\beta\delta\beta + \theta_\delta\delta\zeta$
$\bar{\theta}$	=	equilibrium pitch angle $\theta_o \pm \theta_s S\psi + \theta_c C\psi + \theta_\beta(\bar{\beta}-\beta_{pc})$, rad
θ_β, θ_ζ	=	pitch-flap and pitch-lag coupling ratios
λ	=	inflow ratio $(V_i+V_z)/\Omega R$
Λ	=	complex eigenvalue, $\Lambda = \eta+i\omega$
μ	=	advance ratio, $V_x/\Omega R$
ρ	=	air density slugs/ft^3
σ	=	rotor solidity, $bc/\pi R$
T	=	period of one revolution, $T = 2\pi/\Omega$ sec
$\bar{\Phi}$	=	inflow parameter, $\bar{\Phi} = 4/3\lambda$
ψ	=	rotor azimuth angle, $\psi = 0$ aft, $\psi = \Omega t$, rad, dimensionless time
ω	=	imaginary portion of lead-lag eigenvalue
ω_β, ω_ζ	=	dimensionless nonrotating flap and lag frequencies at $\theta = 0$
Ω	=	rotor angular velocity
$S\psi$, $C\psi$	=	sine (ψ), cosine (ψ)
$(\dot{\ })$	=	$\frac{d}{d\psi}(\ ) = \frac{1}{\Omega}\frac{d}{dt}(\ )$

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